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ON THE ASYMPTOTIC THEORY OF LAMINAR SEPARATION ON A MOVING SURFACE*

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The steady flow of an incompressible fluid is considered in the neighbourhood of the point of boundary layer separation on a rigid surface moving downstream. It is shown that separation occurs in the region downstream of the interaction region. The pressure distribution in that region is obtained from the solution of the boundary value problem for equations of the boundary layer with a given displacement thickness.

Consider the plane steady flow of a viscous incompressible fluid in the neighbourhood of the separation point on a surface that moves downstream at constant velocity. By the asymptotic theory /1/ there is in the neighbourhood of the separation point at high Reynolds numbers an interaction region, where a large unfavourable selfinduced pressure gradient is acting. Upstream of that region the flow is defined by the Prandtl equations. Outside that region (i.e. in the body scales) the pressure distribution is determined by the solution of the potential flow theory of a perfect fluid with free streamlines.

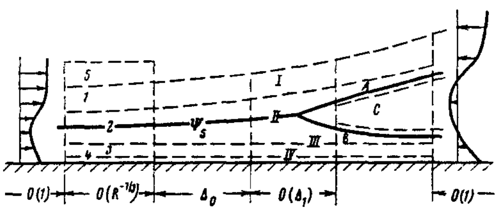

Fig.1

We will present some of the results obtained in /1/ related to the solution of the problem of the interaction region, which will be necessary subsequently.

We will use the following notation: Lx, Ly are the curvilinear orthogonal coordinates directed along the body surface and normal to it, respectively, $U_{00}u$, $U_{00}v$ are the projections of the velocity vector along these axis, $p_{00} + \rho U_{00}^2 p$ is the pressure, and $U_{00}L\psi$ is the stream function. Here L is a characteristic dimension of the body, U_{00} and p_{00} are the velocity and pressure along a free streamline, and ρ is the density. The Reynolds number $R = U_{00}L/\nu$ as $R \to \infty$. For convenience we set the flow separation point longitudinal coordinate equal to zero in the body scales, and denote the surface velocity of the body by $U_{00}U_w$ $(U_w > 0,\ U_w = O(1))$.

In the interaction region whose longitudinal dimension $x = O(R^{-1/3})$, the solution of the Navier-Stokes equations has a multilayer structure, In the basic part of the boundary layer and in the region adjoining the body surface (regions 1 and 3 in Fig.1) the longitudinal component of the velocity vector is of the order of unity, and the solution (in Mises variables) has the form

$$
\begin{aligned}
& x = R^{-1/3}x^*,\ \psi = R^{-1/2}\Psi \\
& u = U_i(\Psi) + R^{-1/6}U_i^*(x^*, \Psi) + O(R^{-1/3}\ln R) \\
& y = (6a_0)^{-1}R^{-1/2}\ln R + R^{-1/2}[G_1(x^*) + Y_1(\Psi)] + \\
& \quad O(R^{-2/3}\ln R) \\
& y = R^{-1/2}Y_3(\Psi) + O(R^{-2/3}) \\
& p = R^{-1/6}p_i(x^*, \Psi) + O(R^{-1/3}\ln R) \\
& \partial p_i/\partial\Psi = 0,\ \ p_i(x^*, \Psi) = p^*(x^*) \\
& U_i^* = -p^*(x^*)\,U_i^{-1}(\Psi) + B^*u_i(\Psi) \\
& Y_i(\Psi) = \int U_i^{-1}(\Psi)\,d\Psi,\ \ U_1(\infty) = 1 \\
& G_1(x^*) = -a_0^{-1}\ln[-p^*(x^*) + B^*] + b_0 + a_0^{-1}\ln(2a_0^2) + \\
& \quad \Phi(x^*) \\
& \Phi(x^*) \equiv 0\ (x^* < 0)
\end{aligned}
\tag{1.1}
$$

where the subscript $i = 1, 3$ relates to regions 1 and 3, respectively, the constants a_0 and b_0 define the profile of the boundary layer at the point $x = -0$, and the meaning of the constant B^* and the function $\Phi(x^*)$ is explained below.

Between these two regions in the neighbourhood of the streamline $\Psi = \Psi_s$ is the region of low speeds (region 2) that makes the main contribution to the displacement action of the

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boundary layer. Owing to the considerable local pressure gradient, the flow is inviscid and the solution may be represented in the form

$$\begin{aligned}
&\psi = R^{-1/2}\Psi_s + R^{-9/16}\Psi^* \\
&u = R^{-1/16}u^*(x^*, \Psi^*) + O(R^{-1/8}\ln R) \\
&y = (12a_0)^{-1}R^{-1/2}\ln R + R^{-1/2}[G_2(x^*) + y^*(x^*, \Psi^*)] + \\
&\quad O(R^{-5/8}\ln R) \\
&u^* = [a_0^2\Psi^{*2} - 2p^*(x^*) + 2B^*]^{1/2} \\
&y^* = \frac{1}{2a_0}\ln\frac{u^*(x^*, \Psi^*) + a_0\Psi^*}{u^*(x^*, \Psi^*) - a_0\Psi^*} \\
&G_2(x^*) = (1/2)[G_1(x^*) + \Phi(x^*) + b_0] \\
&\Phi(x^*) \equiv 0 \ (\Psi^* < 0)
\end{aligned} \tag{1.2}$$

Immediately adjacent to the body surface is the viscous part of the boundary layer (region 4), but its importance in the interaction process is secondary, and hence its solution is not presented here.

The displacing action of the boundary layer on its external boundary, as follows from (1.1), is determined by the equation

$$G_1'(x^*) = \frac{-p^{*\prime}(x^*)}{a_0[p^*(x^*) - B^*]} + \Phi'(x^*) \tag{1.3}$$

and provides the relation between the slope of the streamlines and pressure distribution. The second equation that gives this relation and closes the problem is defined by the integral of the theory of small perturbations, which defines the flow in region 5 which includes the external potential flow and has the transverse dimension $y = O(R^{-1/16})$.

The solution of the problem for the interaction region contains an arbitrary constant B^* that is the additive constant of the Bernoulli function for region 2. A solution for $B^* > 0$ was obtained in /1/ (an imaginary solution is obtained for $B^* < 0$). It was found that inside the boundary layer at point $x^* = 0, \Psi^* = 0$ (in region 2) a bifurcation of the streamline occurs, and that beyond that point there is a region of reverse flow in which the pressure variation is a quantity of the order of $R^{-1/8}$, and the function $\Phi(x^*)$ then defines the form of the upper separated streamline $\Psi^* = 0$. However, nowhere in region 2 does the lengthwise component of the velocity vector vanish. Hence it was assumed that reverse flow (i.e. the satisfaction of the condition $u = \partial u/\partial y = 0$) in conformity with the Moore-Rott-Sears criterion /2/) occurs in the inner viscous regions imbedded in region 2. Attempts to construct a solution for these regions met with considerable difficulties. But, if we set $B^* = 0$ and stipulate the appearance of the reverse flow in region 2, a constradiction occurs.

On the other hand, as shown in /4/, if we assume $B^* = 0$ (and, consequently, also $\Phi(x^*) = 0$) and do not stipulate the occurrence of reverse streams, contradictions do not arise. The integral of small-perturbation theory together with (1.3) yields the equation

$$\begin{aligned}
&A(x_1) = -\frac{p_1'(x_1)}{2p_1(x_1)} = x_1^{1/2}H(x_1) - \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{p_1(t) + (-t)^{1/2}H(-t)}{t - x_1}\,dt \\
&x^* = (a_0k)^{-2/3}x_1, \quad p^*(x^*) = 2k^{2/3}a_0^{-1/3}p_1(x_1)
\end{aligned} \tag{1.4}$$

where $H(x_1)$ is the Heaviside function and the positive constant k depends on the position of the separation point in the external potential flow. The position of that point is determined by the solution in a region of the order of the dimensions of the body and the integral is understood in the sense of its principal value.

From the merging with the solution outside the interaction region it follows that

$$\begin{aligned}
&p_1(x_1) = -(-x_1)^{1/2} + \lambda_0(-x_1)^{-1/2} + O[(-x_1)^{-3/2}] \\
&A_1(x_1) = [4(-x_1)]^{-1} + O[(-x_1)^{-2}] \ (x_1 \to -\infty) \\
&A_1(x_1) = x_1^{1/2} + \lambda_0 x_1^{-1/2} - (4x_1)^{-1} + O(x_1^{-3/2}) \\
&(x_1 \to \infty)
\end{aligned} \tag{1.5}$$

The constant λ_0 in (1.5) cannot be determined from the solution of the local problem, and requires a solution of the problem for the second term of the expansion in R in the region $y = O(1), x = O(1)$. To be specific we put $\lambda_0 = 0$. On passing to the solution with $\lambda_0 \neq 0$, a shift occurs in the system of coordinates, so that x becomes $x - 2\lambda_0(ka_0)^{-2/3}R^{-1/16}$. The solution of problem (1.4), (1.5) when $\lambda_0 = 0$ is shown in Fig.2 (see /4/).

From formulas (1.1)–(1.5) as $x^* \to \infty$ we have

$$\begin{aligned}
&p^*(x^*) \to -x^{*1/2}\exp(-\gamma_0 x^{*3/2}) \\
&G_1(x^*) \to (4/3)kx^{*3/2} - (2a_0)^{-1}\ln x^* + b_0 + a_0^{-1}\ln(2a_0^2) + o(1) \\
&\gamma_0 = 4ka_0/3
\end{aligned} \tag{1.6}$$

(henceforth $B^* = \Phi(x^*) = 0$ everywhere).

Thus it follows from (1.6) and (1.2) that the interaction only leads to further slowing down of the flow in region 2, hence the separation point must lay in the region downstream of it.

2. To obtain a solution that would define the separation mechanism we shall consider the flow in region II that is the continuation of the slow flow region 2 and lies beyond the interaction region.

When $x^* \to \infty$, the decrease in the pressure gradient in (1.6) must result in the Bernoulli equation being no longer applicable in region II owing to the action of internal friction forces. (This also follows from a consideration of subsequent terms of the expansion for the interaction region.) The pressure variations that occur in this case cannot result in a change in the slope of streamlines on the external boundary layer, since otherwise this region would be the same as the region where $x^* = R^{1/3}x = O(1)$.

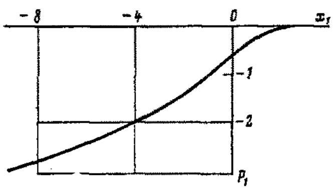

Fig.2

The exponential approach of the pressure to zero as $x^* \to \infty$ indicates that in region II $p^* = \sigma p_0 = R^{1/6}p$, where the small parameter $\sigma = \sigma(R) \to 0$ as $R \to \infty$. If now, instead of x^*, we take as the indpendent variable p^*, then from (1.6) we obtain that in region II

$$x = R^{-1/3}x^* = \Delta_0 + \Delta_1 X(p_0) + o(\Delta_1) \qquad (2.1)$$
$$\Delta_0 = \gamma_0^{-2/3}R^{-1/3}(\ln\sigma^{-1})^{2/3} + (2/9)\gamma_0^{-2/3}R^{-1/3}(\ln\ln\sigma^{-1})(\ln\sigma^{-1})^{-1/3}$$
$$\Delta_1 = R^{-1/3}(\ln\sigma^{-1})^{-1/3}$$

From the merging with (1.6) as $p_0 \to -\infty$, we have

$$X(p_0) = -\frac{2}{3}\gamma_0^{-2/3}\ln(-\gamma_0^{1/3}p_0) + o(1)$$

the value of σ remains so far undetermined. The expansion of x obtained indicates that region II, in which the separation point lies, is at a distance Δ_0 from the interaction region (Fig.1).

Using (1.1), (1.2), (1.6), and the expressions for x and changing to the original variables, we can represent the solution for region II in the form

$$x = \Delta_0 + \Delta_1 X,\ \psi = R^{-1/2}\Psi_s + \sigma^{3/4}R^{-7/12}\Psi_0 \qquad (2.2)$$
$$u = \sigma^{1/2}R^{-1/12}u_0(X, \Psi_0) + o(\sigma^{1/2}R^{-1/12})$$
$$p = \sigma R^{-1/6}p_0(X, \Psi_0) + o(\sigma R^{-1/6})$$
$$y = R^{-1/2}[(12a_0)^{-1}\ln R + (2a_0)^{-1}\ln\sigma^{-1} + b_0 + a_0^{-1}\times$$
$$\ln(2a_0^2\gamma_0^{1/3}) + y_0(X, \Psi_0)] + o(R^{-1/2})$$

Substituting expansion (2.2) together with (2.1) into the Navier-Stokes equations and using (on the basis of above considerations) the balance of inertial and viscous terms with the pressure gradient, we obtain the equations of the boundary layer

$$u_0\frac{\partial u_0}{\partial X} + \frac{\partial p_0}{\partial X} = u_0\frac{\partial}{\partial\Psi_0}\left(u_0\frac{\partial u_0}{\partial\Psi_0}\right) \qquad (2.3)$$
$$\frac{\partial p_0}{\partial\Psi_0} = 0, \quad \frac{\partial y_0}{\partial\Psi_0} = \frac{1}{u_0}$$

For the small parameter we then obtain the transcendental equation

$$\sigma(\ln\sigma^{-1})^{2/3} = R^{-1/3} \qquad (2.4)$$

Since region 2 and II do not overlap, it is necessary to merge them in the variables p, ψ as $p^* \to -0$ and $p_0 \to -\infty$. Then for the solution in region II as $X \to -\infty$ (i.e. in the original variables) we obtain

$$u_0 \to \gamma_0^{1/3}\exp(-\gamma_1 X)F_0(\tau) \qquad (2.5)$$
$$p_0 \to -\gamma_0^{-1/3}\exp(-2\gamma_1 X),\ y_0 \to \gamma_1 a_0^{-1}X + H_0(\tau)$$
$$\tau = \gamma_0^{1/6}\Psi_0\exp(\gamma_1 X),\ F_0(\tau) = (a_0^2\tau^2 + 2)^{1/2}$$
$$H_0(\tau) = (2a_0)^{-1}\ln\frac{F_0(\tau) + a_0\tau}{F_0(\tau) - a_0\tau} - (2a_0)^{-1}\ln(2a_0^2\gamma_0^{1/3})$$
$$\gamma_1 = (3/4)\gamma_0^{2/3}$$

In the basic part of the boundary layer $X = O(1)$ (region I) which is the continuation of region 1, it is necessary to take into account the transverse pressure change associated with the streamline curvature. Hence using (1.1) and (1.6) (taking into account, generally speaking, the next terms of expansion in the solution for the interaction region), we obtain

$$u = U_1(\Psi) + O(\Delta_0), \quad p = \Delta_1 p_1^\circ(X, \Psi) + o(\Delta_1)$$
$$y = R^{-1/8}[(6a_0)^{-1}\ln(R\sigma^{-6}) + Y_1(\Psi) + g_1(X)] + o(R^{-1/8})$$

Substituting this expansion into the Navier-Stokes equations, we find that $g_1''(X) = 0$. (Other expressions and the next terms of the expansion are not subsequently used and are not presented here). Note that the terms of the expansion for the pressure depend on Ψ, but all of them up to the term $O(\sigma R^{-1/4})$ approach zero as $\Psi \to \Psi_s$.) Taking into account (2.5), from

the merging with the solution in region II, we obtain that $g_1(X) = 2\gamma_1 a_0^{-1}X + a_0^{-1}\ln(2a_0^2\gamma_0^{1/3}) + b_0$ also as $\Psi_0 \to \infty$

$$u_0 \to a_0\Psi_0, \quad y_0 \to a_0^{-1}\ln\Psi_0 + 2\gamma_1 a_0^{-1}X + o(1) \tag{2.6}$$

The last formula defines the displacement thickness on the external boundary of region II.

In the lower part of the boundary layer when $X = O(1)$ (region III) the solution is similar to the solution in region I, the only difference being that there is no change in transverse pressure in the principal terms. The solution has the form

$$u = U_3(\Psi) + O(\Delta_0), \quad p = \sigma R^{-1/4}p_0(X) + o(\sigma R^{-1/4})$$
$$y = R^{-1/2}Y_3(\Psi) + o(R^{-1/2})$$

As shown in /1/, the expressions for functions $U_3(\Psi)$, $Y_3(\Psi)$ are

$$U_3(\Psi) = a_0(\Psi_s - \Psi) + O[(\Psi_s - \Psi)^3]$$
$$Y_3(\Psi) = -a_0^{-1}\ln(\Psi_s - \Psi) + b_* + O[(\Psi_s - \Psi)^2]$$
$$(\Psi \to -\Psi_s)$$

and, then, from the merging with the solution in region II follows that as $\Psi_0 \to -\infty$

$$u_0 \to a_0(-\Psi_0), \; y_0 \to -a_0^{-1}\ln(-\Psi_0) - a_0^{-1}\ln(2a_0^2\gamma_0^{1/3}) + o(1) \tag{2.7}$$

Let us investigate the behaviour of the solution in region II as $X \to \infty$. Formulas (2.6) and (2.7) show that as $X \to \infty$ the region of reverse flow expands linearly. In the separated and the lower parts of the boundary layer, as the principal terms, the velocity profiles $U_i(\Psi)$ are maintained, and mixing layers (regions A and B) develop between the reverse flow region (region C) and regions I and II. Then, as $X \to \infty$ we obtain the following solutions for the lower and upper parts of region II (the plus and minus signs correspond to these, respectively) /3/:

$$u_0 \to Xf_+(\eta), \quad y_0 \to 2\gamma_1 a_0^{-1}X + a_0^{-1}\ln X + h_+(\eta) \tag{2.8}$$
$$f_+(\eta) = a_0\eta + a_0^2, \quad h_+(\eta) = a_0^{-1}\ln(\eta + a_0), \quad \eta = \Psi_0/X$$
$$u_0 \to Xf_-(\xi), \quad y_0 \to -a_0^{-1}\ln X - a_0^{-1}\ln(2a_0^2\gamma_0^{1/3}) +$$
$$h_-(\xi), \quad f_-(\xi) = a_0\xi + a_0^2$$
$$h_-(\xi) = -a_0^{-1}\ln(\xi + a_0), \quad \xi = -\Psi_0/X$$

For the reverse flow as $X \to \infty$ we obtain

$$u_0 \to -a_0^2\gamma_1^{-1}, \quad p_0 \to -a_0^4\gamma_1^{-2}/2 \tag{2.9}$$
$$y_0 \to a_0^{-2}\gamma_1 X(a_0 - \Psi_0/X) - (2a_0)^{-1}\ln(2a_0^2\gamma_0^{1/3})$$

Formulas (2.8) and (2.9) close the system of boundary conditions for region II.

3. Carrying out the transformation

$$X = \gamma_1^{-1}X', \quad y_0 = a_0^{-1}Y' + a_0^{-1}\ln(a_0\gamma_1^{-1}) \tag{3.1}$$
$$u_0 = a_0^2\gamma_1^{-1}u', \quad p_0 = a_0^4\gamma_1^{-2}p', \quad \Psi_0 = a_0\gamma_1^{-1}\Psi'$$

we write Eqs. (2.3) together with boundary conditions (2.5)–(2.9) in conventional variables (omitting the primes)

$$\frac{\partial\Psi}{\partial Y}\frac{\partial^2\Psi}{\partial X\,\partial Y} - \frac{\partial\Psi}{\partial X}\frac{\partial^2\Psi}{\partial Y^2} + \frac{dp}{dX} = \frac{\partial^3\Psi}{\partial Y^3} \tag{3.2}$$
$$\Psi \to \exp(Y - 2X) \quad (Y \to \infty)$$
$$\Psi \to -(2\alpha_0)^{-1}\exp(-Y) \quad (Y \to -\infty)$$
$$\Psi \to \left(\frac{2}{\alpha_0}\right)^{1/2}\exp(-X)\,\mathrm{sh}\left[Y - X + \frac{1}{2}\ln(2\alpha_0)\right]$$
$$p \to -\alpha_0^{-1}\exp(-2X) \quad (X \to -\infty)$$

$$\Psi \to X\varphi_+(\eta_+), \quad \varphi_+(\eta_+) = \exp \eta_+ - 1$$
$$\Psi \to X\varphi_-(\eta_-), \quad \varphi_-(\eta_-) = 1 - \exp(-\eta_-)$$
$$\eta_+ = Y - 2X - \ln X, \quad \eta_- = Y + \ln(2\alpha_0 X)$$
$$\Psi \to X(1 - Y/X) - \ln(2\alpha_0)^{1/2}, \quad p \to -{}^1/_2 \quad (X \to \infty)$$
$$\alpha_0 = 4a_0^3/(3k)$$

The solution of the problem thus depends on the single parameter α_0. The appearance in it of positive constants a_0 and k are determined from the global solution; to the different α_0 there then correspond different positions of the separation point in region II.

The pressure distribution in problem (3.2) must be determined by the solution, while the displacement thickness is specified. Boundary value problems of this type for equations of the boundary layer appeared in /5—8/. Unlike the classical approach, in which the pressure gradient is specified, solutions of these problems behave consistently at the point of zero friction. Hence also for (3.2) at the separation point where in conformity with the Moore-Rott-Sears criterion $\Psi_Y = \Psi_{YY} = 0$, the irremovable singularity /9/ does not arise.

Note that, using the transposition,

$$Y = Z + X - \ln(2\alpha_0)^{1/2}$$

problem (3.2) reduces to a form that is symmetrical about the $Z = 0$ axis, and hence the conditions at $-\infty$ can be replaced by the conditions $\Psi = \Psi_{ZZ} = 0$ when $Z = 0$.

4. Let us briefly consider the flow beyond the region $X = O(1)$ in order to show that the solution obtained becomes the solution for the region with the longitudinal scale of the order of body dimensions $(x = O(1))$ as $x \to +0$.

Region I-III are situated at a distance $O(\Delta_0)$ (see (2.1)) from the origin of coordinates, hence to obtain the solution when $x \to 0$, it is necessary to consider regions in which

$$x = \gamma_0^{-3/2} R^{-1/2} (\ln \sigma^{-1})^{3/2} + R^{-1/2} (\ln \ln \sigma^{-1}) (\ln \sigma^{-1})^{-1/2} X_1, \quad X_1 > 2/(9\gamma_0^{3/2})$$

and

$$x = R^{-1/2} (\ln \sigma^{-1})^{3/2} X_2, \quad X_2 > \gamma_0^{-3/2}$$

In region A, which is a continuation of the upper viscous part of region II, the solution in conformity with (2.8), when $X_2 = O(1)$, can be represented in the form

$$\Psi_0 = (\ln \sigma^{-1}) \Psi_2, \quad u = \Delta_1 (\ln \sigma^{-1}) (X_2 - \gamma_0^{-3/2}) \Phi_+(s) + o(\Delta_1 \ln \sigma^{-1}), \quad y = R^{-1/2} [(12a_0)^{-1} \ln R + 4kX_2^{3/2}/3 - (2a_0)^{-1} \ln \sigma^{-1} + 2(3a_0)^{-1} \ln \ln \sigma^{-1} + a_0^{-1} \ln (X_2 - \gamma_0^{-3/2}) - (2a_0)^{-1} \ln X_2 + H_+(s) + b_0 + a_0^{-1} \ln (2a_0^2)] + o(R^{-1/2}) \tag{4.1}$$
$$s = \Psi_2 (X_2 - \gamma_0^{-3/2})^{-1}, \quad \Phi_+(s) = a_0 s + a_0^2$$
$$H_+(s) = a_0^{-1} \ln (s + a_0)$$

In the basic part of the detached boundary layer the solution that satisfies the conditions of merging with (4.1) has the form

$$u = U_1(\Psi) + R^{-1/2} (\ln \sigma^{-1})^{3/2} u_1^0 (X_2, \Psi) + o[R^{-1/2} (\ln \sigma^{-1})^{3/2}], \tag{4.2}$$
$$p = \Delta_1 p_2 (X_2, \Psi) + o(\Delta_1)$$
$$y = R^{-1/2} \left[(6a_0)^{-1} \ln R + \frac{4}{3} kX^{3/2} \ln \sigma^{-1} - (3a_0)^{-1} \ln \ln \sigma^{-1} - (2a_0)^{-1} \ln X_2 + Y_1(\Psi) + b_0 + a_0^{-1} \ln (2a_0^2) \right] + o(R^{-1/2})$$
$$\frac{\partial u_1^\circ}{\partial X_2} = -2k^2 U_1 + (U_1 U_1')', \quad p_2 = -kX_2^{-1/2} \int U_1(\Psi)\, d\Psi$$

As $X_2 \to \gamma_0^{-3/2}$ the solution (4.1), (4.2) merges with the solution in region $X_1 = O(1)$ as $X_1 \to \infty$, where it is similar to (4.1), (4.2) and represents in essence the solution for regions I and II expressed in variables $X_1 = O(1)$, taken as $X \to \infty$.

In the lower part of the boundary layer the solution is similar to (4.1), (4.2).

The solution (4.1), (4.2) merges with the solution for the region $x = O(1)$ as $x \to 0$ and $X_2 \to \infty$, and, as will be readily seen, the basic part of the boundary layer and region A becomes a mixing layer with characteristic transverse dimension of the order of $R^{-1/2}$. Region B and the boundary layer lower part (together with boundary layer IV) become the conventional boundary layer on the moving surface with zero velocity on the external boundary. The reverse flow (region C) depends on the ejection of the detached and lower parts of the boundary layer. Here $p = O(\sigma R^{-1/4})$ as $X_2 \to \infty$, and $\sigma^{-1} R^{1/4} p \to -9a_0^2 (8k^2 X_2)^{-1}$, i.e. by virtue of (2.4), as $x \to 0$ the variable part of the pressure is the quantity $O(R^{-1})$.

The solution obtained obviously defines a particular form of unsteady flow in which the separation point moves upstream at constant velocity $U_s = -U_w$.

Note that the destruction of the trail occurs /10/, as also does separation on a moving surface, in the viscous region that lies behind the interaction region. The solution in that region reduces to a problem of the form (3.2), where the conditions as $Y \to -\infty$ are replaced by the conditions at the trail axis of symmetry $\Psi = \Psi_{YY} = 0$ when $Y = 0$.

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A THREE-DIMENSIONAL HYPERSONIC VISCOUS SHOCK LAYER IN TWO-PHASE FLOW*

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A three-dimensional hypersonic flow of visous gas containing solid or liquid deformable particles past smooth blunt bodies with permeable surfaces, is considered. A numerical solution is obtained near the stagnation point of double curvature for a wide range of values of the Reynolds number, sizes and compositions of the particles, shape of the body and the injection (suction) parameters. Characteristic velocity and temperature profiles across the shock layer are given for each phase, and also the dependence of the separation, friction and heat exchange coefficients at the body surface on the Reynolds number and other defining parameters of the problem. It is shown that the presence of particles in the flow leads, other conditions being equal, to a reduction in the separation of the shock wave. The asymptotic behaviour of the equations of the three-dimensional two-phase hypersonic shock-layer is analysed for the limiting case of small particles. It is shown that in this case the flow separates into two layers; equations are given for the principal terms of the expansions in each layer, and boundary conditions are given following from the conditions for matching the solutions in adjacent regions. An analytic solution of the problem in the approximation of two inviscid layers separated by a contact surface is obtained for the layer adjacent to the body near the stagnation point for large Reynolds numbers and strong injection.

The motion of heterogeneous particles in plane or axisymmetric shock layers was studied earlier in /1/, in the inviscid formulation and assuming that the effect of the particles on the gas-dynamic parameters is small. A numerical solution of the problem of a supersonic, inviscid two-phase flow past a sphere was obtained in /2—4/. Homogeneous gas flow in a viscous, hypersonic three-dimensional shock layer near the stagnation point was studied in /5/.

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